APPLICATION OF FINITE DIFFERENCE METHOD(FDM) AND FINITE ELEMENT METHOD(FEM) IN SOLVING HEAT PROBLEM

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ABSTRACT

The numerical methods known as the Finite Difference Method and the Finite Element Method are utilized widely for solving heat problems, particularly in engineering applications. This research aims to determine the optimal approach for solving 2D regular geometry heat transfer, comparing between two methods which are the Finite Difference Method (FDM) and the Finite Element Method (FEM). The investigation focuses on steady-state heat transfer scenarios. MATLAB is employed for computations and generating temperature distribution graphs. The findings suggest that the Finite Element Method is more accurate in addressing 2D steady-state heat problems, as evidenced by a larger blue region in the error distribution plot.

Keywords: 2D regular geometry, heat transfer, finite difference method, finite element method

1.0 INTRODUCTION

An approximation of the answers to a wide range of scientific and technical problems can be found by a methodology known as a numerical approach. The vast majority of practical engineering issues do not have analytically solvable solutions [1], which is the primary reason why numerical methodologies are required to solve them.

The Finite Difference Method (FDM) and Finite Element Method (FEM) are two approximation techniques commonly employed for solving partial differential equations (PDEs). They have been applied extensively to tackle a diverse range of problems, which may encompass linear or nonlinear, time-independent or dependent scenarios. These methods are versatile, capable of addressing problems with various boundary configurations, boundary conditions, and components. Despite the early recognition of the concept by Gauss and Boltzmann, it wasn't until the 1940s [2] that these methods gained widespread adoption for resolving engineering challenges. The Finite Difference Method (FDM) is one way to turn the governing partial differential equations of a heat transfer system into numerical solutions. This can be done in several different ways. Approximation, in which finite differences are substituted for partial derivatives, is the method that is used to achieve this goal. This is the value that can be found at each individual grid point within the domain.

The finite element method (FEM) also can be used to solve heat transfer problems. It is well-known that FEM can handle complex boundary value problems [3]. Experimental and analytical methodologies are necessary during the validation phase of the numerical methodology.

The heat conduction problem was presented as:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+U=0$$

where T represents temperature and U represents an equation for heat source [4].

The heat equation is one of the most significant partial differential equations that represents the distribution of heat or the change in temperature over time in a specific location. Using numerical methods like the Finite Difference Method and Finite Element Method [5] the two-dimensional heat equation can be solved both theoretically and numerically.

2.0 LITERATURE REVIEW

The transport problem that involves heat conduction and mass-diffusion, which is a typical time-dependent problem, is something that always occurs in nature, and particularly in the engineering area [6]. This problem falls under partial differential problem (PDE). Under certain initial conditions, the application of basic geometries and boundary conditions may produce analytical findings. It is necessary to study heat equations to properly analyze the behaviors of the components that are involved in any project [7]. The equation solutions are directly influenced by the imposed conditions, as well as the initial conditions and the boundary conditions. To solve problems involving complex geometries and boundary conditions whereby an analytical solution is challenging or impossible to obtain, numerical methods played an essential role [8]. The Fourier law, which may be written as partial differential equations, can be used to determine the conductive heat flow at any location within the interior or surface of a body (PDEs).

The Finite Differences Method (FDM), which is easy to read and operate, and the Finite Element Method (FEM) are both examples of the several approaches that can be taken to address issues pertaining to heat transmission. The FDM is a numerical method for solving PDEs that involves discretizing a continuous physical domain into a finite discrete mesh and then approximating each partial derivative in the PDE using finite algebraic differences. The solution to the PDE's finite difference equation must precisely represent every point in the discretized region where the solution is required. The FEM provides a systematic and general method for modelling solutions to approximation boundary problems, such as the Galarkin’s technique [9]. Approximate solutions to differential equations with a boundary condition can be generated by partitioning the solution domain into a finite number of subdomains. Each element will be calculated using a subset of the mesh generation and conversion PDE problem before being reassembled as nodes in the original domain. For 2D geometry, the generated mesh can take on a variety of different shapes, including triangles, rectangles, and quadrilaterals [3].

The term "heat transfer" refers to the study of the transmission of heat energy from one location to another through chemical interaction, fluid motion, and electromagnetic waves [5]. The distribution of heat or the change in temperature over time is described by the heat equation, a crucial partial differential equation [10]. The heat equation in two dimensions can be solved in practice both theoretically and numerically. Since heat problem is partial differential problem, it categorizes in parabolic, hyperbolic, and elliptic. The model of elliptic PDE is steady state behavior, in this case time is not the factor. One of the examples is steady state heat distribution on the regular plates that every edge has its own temperature [11].

The finite difference method is useful for solving heat transport problems (FDM). However, due to the method's inherent imprecision [1], this methodology will lead to some inconsistencies in the findings. While finite difference methods (FDM) are simpler to compute and implement, they are inadequate for solving problems with complicated geometries in all cases. Taylor expansion is used to approximate the solution of PDE when FDM is implemented. The discretization of PDE is constructed using a network of lines in a regular pattern [12]. The idea of FDM is based on derivative at a point replaced by a difference quotient over a small interval while FEM dividing the system into small pieces or called as finite element and use element equations to assemble the element to represents the original system [13]. Due to its ability to minimize an error function and generate a stable solution, finite element analysis (FEM) is the superior method for dealing with the difficult problem.

3.0 PROBLEM STATEMENT

The heat transfer problem revolves around establishing the temperature distribution within a defined domain, while adhering to specific boundary conditions and heat sources. This domain is delineated by a series of boundaries, and the temperature distribution within it must be ascertained at discrete points. Typically, the FDM and the FEM serve as the primary numerical approaches for resolving partial differential equations, which involve heat conduction problems. Consequently, addressing the heat transfer problem requires the implementation of these methods to determine the temperature distribution effectively.

4.0 SIGNIFICANCE OF RESEARCH

The findings of this study can be applied to future investigations in fields that are closely related to them. This research will give a clearer vision of FDM and FEM, where it will eventually lead to deeper understanding on utilizing the FDM and FEM in the process of solving associated problems, particularly in the domains of science and engineering to select the suitable method and accurately. This is one of the goals of this research. The findings help to improve the accuracy of heat transfer simulations by determining the method that give the reliable result. Because of this, it is possible for it to assist fields that are closely connected, such as engineering, to be employed in the investigation. Therefore, it offers educational value by practical demonstration of FDM and FEM in the context of heat conduction problems.

5.0 RESEARCH METHODOLOGY

The research steps are clearly described one by one in the form of one step each. Therefore, the research methodology shown clearly by using the flowchart shows in Figure 1 below:

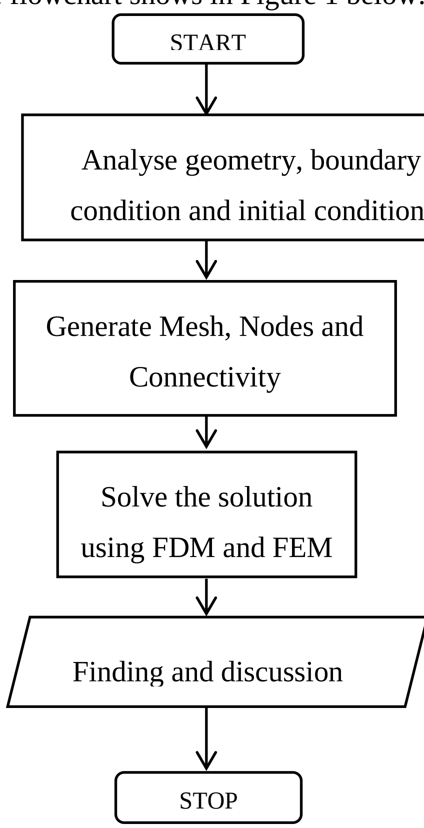

Figure 1: Flowchart of the Research Method

6.0 RESULT AND DISCUSSION

This article will consider two problems of heat transfer which are:

Problem 1

The steady state two-dimensional heat conduction problem presented in Figure 4.2.1 in a square shaped is governed by the following PDE and the associated initial and boundary conditions:

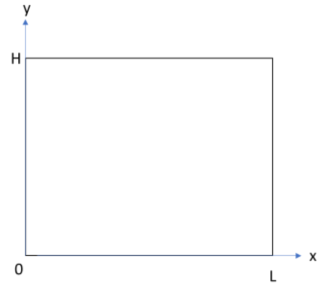

Figure 2: Square shaped governed by PDE

$$L \times H = (0.1m \times 0.10m)$$

The initial & Boundary Conditions are:

Initial Condition: $T(x, y) = 30°C$

Boundary Conditions: $T(0, y) = 500°C;$

$$T(L, y) = 45°C$$

$$T(x, 0) = 500°C;$$

$$T(x, H) = 45°C$$

Problem 2

Consider steady two-dimensional heat transfer in a square cross section (3cm X 3cm) with the prescribed temperatures at top, right, bottom, and left surfaces to be $100℃, 200℃, 300℃ \; and \; 500℃$, respectively.

For the next steps, from these two problems solved by using the MATLAB to compute the error distribution between FDM and FEM.

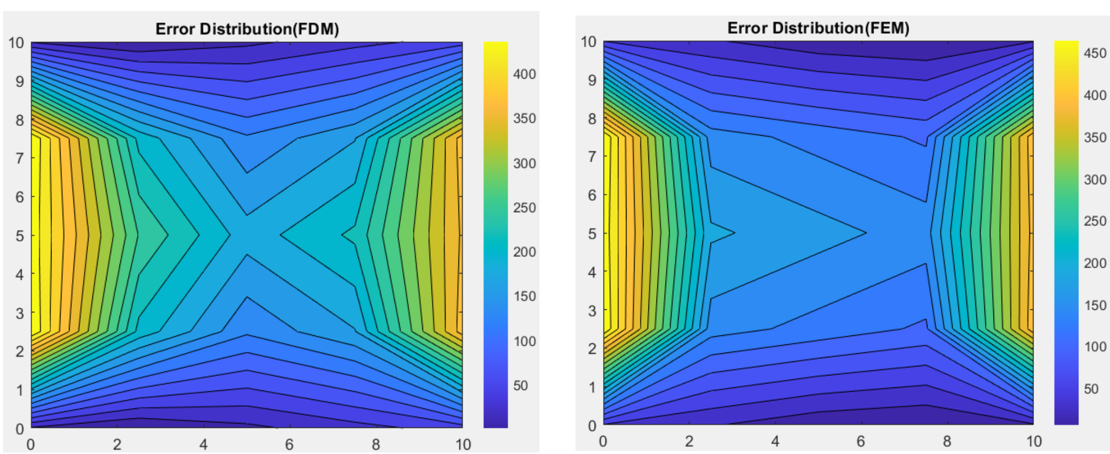

Figure 3: Error Distribution of FDM and FEM for Problem 1

The result shown above is from the MATLAB output to discover which one from two methods is the best solution or the most accurate. The error distribution can be measured by looking the area of the blue region in the figure which is more area of the blue region is the less error distribution or in other word is FEM is more accurate compared to FDM for the solution of Problem 1.

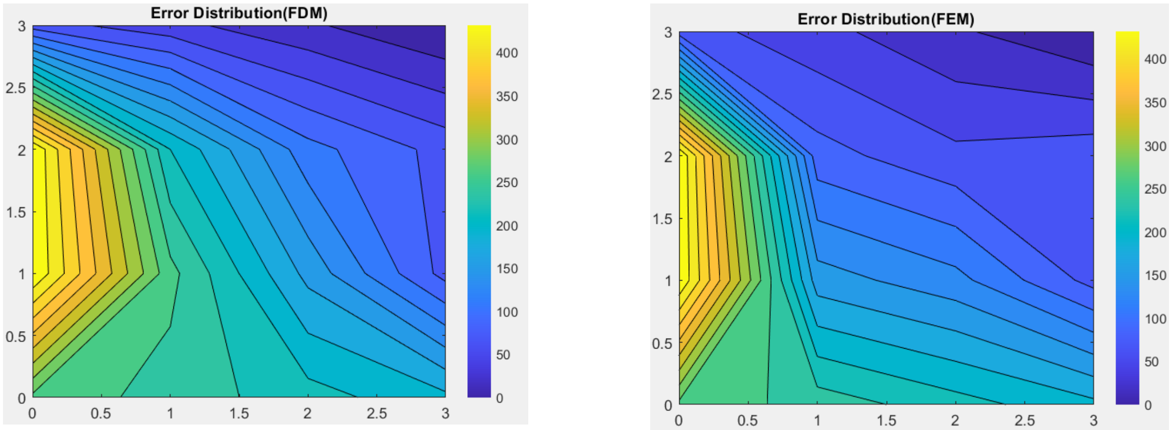

Figure 4: Error Distribution of FDM and FEM for Problem 2

Meanwhile, for the solution of the Problem 2, the result shown above is from the MATLAB output to discover which one from two methods is the best solution or the most accurate. The error distribution can be measured by looking the area of the blue region in the figure which is more area of the blue region is the less error distribution or in other word is FEM is more accurate compared to FDM.

7.0 CONCLUSION AND RECOMMENDATION

Based on articles and other sources that are used in this article, learn a little about how related FDM and FEM are related by knowing the theory of both method before the implementation to solve the problem. Since this article aims to apply the FDM and FEM calculation of both methods has been using MATLAB programming.

For the conclusion, it's difficult to state which method is the best because both methods give the almost accurate solution for 2D steady state heat problem. If compare the step of implementation, can say that Finite difference method is easier to compute as well to write the algorithm in MATLAB than Finite element method, but cannot neglect the result that get from the error distribution that Finite element method is the best method to solve the problems.

For recommendation, to solve more complex geometry heat problems like irregular geometry, complex boundary condition or other factors that related to this topic. By doing these recommendations, researchers can enhance their ability to accurately solve complex geometry heat transfer problems and make informed decisions regarding the choice of method for their specific applications.

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