MJIT 2024 Malaysian Journal of Industrial Technology

SOME PHYSICS APPLICATIONS OF THE NEW INTEGRATION

A.O. Asmaa Omar
Mubayrash, Surman City, Libya Country.
Email: asmaa.mubayrash@sabu.edu.ly

Z.A. Zaineb
Alkawash, Sabratha City, Libya Country.
Email: zaineb.alkawash@sabu.edu.ly

*Corresponding author's email: asmaa.mubayrash@sabu.edu.ly

ARTICLE INFO

Handling Editor: Rahimah Mahat

Article History:
Received 5 September 2024
Received in revised form 7 October 2024
Accepted 23 October 2024
Available online 1 December 2024

Keywords:
Physics, application, integration, differential equations

ABSTRACT

In this work, we apply a new integral transform the Abaoub-Shkheam Transform in solving first and second-order ordinary differential equations with constant coefficients and equations are concerned with a damping mechanical force system and an inductive capacitive electrical circuit in mechanics and electrical circuit problems with initial conditions. Also, we will discuss the applications of the Abaoub-Shkheam Transform in solving the differential equations related to successive radioactive decay of the parent nucleus which is an important subject in nuclear physics. Also included are the formulae for computing the daughter and granddaughter nuclei, as well as instructions for the function of the mechanical system's transfer.

1.0 Introduction

The linear differential equations with constant coefficients find their most important applications in the study of electrical, mechanical, and other linear systems. In fact, such equations play a dominant role in unifying the theory of electrical and mechanical oscillatory systems. In this paper, we are using a new integral transform Abaoub-Shkheam transform [1] to solve mechanics and electrical circuit problems. As Laplace transform, this Abaoub-Shkheam transform is also suitable to obtain the solution of linear non-homogeneous ordinary differential equations with constant coefficients. Abaoub-Shkheam transform to facilitate the process of solving ordinary and partial differential equations in the time domain. This conversion has been applied in many journals, including in Cryptography [4,8] and the solution of ordinary, partial, and integrative differential equations. [6,2,5] This underscores the success and importance of the conversion. This transformation is related to the Laplace, Elzaki, and Mahgoub transformations. We must solve ordinary differential equations in mechanics, electrical circuit difficulties, and mechanical engineering based on the mathematical simplicity of this transform and its fundamental features [10]. solve the problems of mechanics, [13] electrical circuits, and beams problems using Elzaki transform [11] used a new transform – Mahgoub transform to solve the problems of mechanics and electrical circuits. The notion of Laplace transforms is used in science and technology such as electric circuit analysis, control engineering, communication engineering, and nuclear physics, as well as in quantum mechanics to solve differential equations [9,12]. The solutions of differential equations in the successive radioactive decay of the nucleus using Laplace transforms are detailed in this study. When a parent nucleus disintegrates into a daughter nucleus, the daughter nucleus may be radioactive, and the process continues until it reaches a stable nucleus, which is known as successive radioactive disintegration.

2.0 Abaoub- Shkheam Transform "Q – Transform"

The Abaoub-Shkheam transform defined by the integral equations

$$T(u,s) = Q[f] = \int_0^\infty f(ut)\, e^{-\frac{t}{s}}\, dt \qquad (1)$$

provided that the integral exists for some s, where $s \in (-t_1, t_2)$ and $f(t)$ be a function defined for all $t \geq 0$, the Q-transform of $f(t)$ is the function $T(u,s)$ in definition (1) in [1].

If we substitute $ut = y$, then Equation (1) becomes,

$$Q[f(t)] = T(u,s) = \frac{1}{u}\int_0^\infty f(y)\, e^{-\frac{1}{us}y}\, dy \quad (2)$$

2.1. Abaoub-Shkheam transform for some basic functions

Elementary functions include algebraic and transcendental functions [4].

Table (1) The Abaoub-Shkheam Transform for elementary functions

$f(t)$	1	t^n	e^{at}	$\sin at$	$\cos at$	$sinhat$	$coshat$
$Q[f(t)] = T(u,s)$	s	$n!\ u^n s^{n+1}$	$\frac{s}{1 - a\,u\,s}$	$\frac{au\,s^2}{1 + a^2u^2s^2}$	$\frac{s}{1 + a^2u^2s^2}$	$\frac{au\,s^2}{1 - a^2u^2s^2}$	$\frac{s}{1 - a^2u^2s^2}$

The original function $f\,(t)$ in (2) is called the inverse transform or inverse of $T(u,s)$, and is denoted by. $f(t) = Q^{-1}\{T(u,s)\}$, A list of the Q-transforms for elementary functions is presented in Table (1).

2.2. Some properties of Abaoub-Shkheam transform

In this part, we present some properties of the Abaoub-Shkheam transform [4].

i- $Q\{a\,f(t) + b\,g\,(t)\} = a\,Q\{f(t)\} + b\,Q\{g(t)\}$, where a and b are constants.

ii- If $Q\{f(t)\} = T(s,u)$, then $Q\{f(at)\} = \frac{1}{a}T\left(\frac{s}{a}, u\right)$.

iii- $Q\{f^{(n)}(t)\} = \frac{Q\{f(t)\}}{u^n s^n} - \frac{1}{u}\sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{(us)^{n-k-1}}$.

iv- $Q\{f(t) * g(t)\} = u[Q\{f(t)\}\, Q\{g(t)\}]$.

3.0 Applications of "Abaoub – Shkheam" Transform

3.1 Application to Mechanics

Example (1)

Suppose a particle P of mass 2 grams moves on the $X\ axis$ and is attracted towards the origin O with a force numerically equal to $8X$. If it is initially at rest at $X = 10$ find the position at any subsequent time, assuming

a. No other forces act.
b. A damping force numerically equal to 8 times the instantaneous velocity acts.

These problems can be solved by using the "Abaoub – Shkheam" Transform.

From Newton's law, the equation of motion of the particle is

$$\frac{d^2x}{dt^2} + 4x = 0 \quad or \quad 2\frac{d^2x}{dt^2} = -8x \qquad (3)$$

With the initial conditions $x^{(1)}(0) = 0$ and $x(0) = 10$.

Taking the "Abaoub – Shkheam" transform of sides of (3), we have

$$Q\left[x^{(2)}\right] + 4Q[x] = 0$$

Using the "Abaoub – Shkheam" transform for derivatives, then

$$\frac{Q(x)}{u^2s^2} - \frac{1}{u^2s}x(o) - \frac{1}{u}x^{(1)}(0) + 4Q(x) = 0$$

$$Q(x)\left[\frac{1}{u^2s^2} + 4\right] = \frac{1}{u^2s}(10) + \frac{1}{u}(0)$$

$$Q(x) = \frac{10s}{1 + 2^2u^2s^2}$$

Take inverse the "Abaoub – Shkheam" transform, we get

$$x = 10cos2t.$$

1. In this case, the equation of motion of particle is

$$2\frac{d^2x}{dt^2} = -8x - 8\frac{dx}{dt} \quad or \quad \frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0 \qquad (4)$$

with initial conditions $x^{(1)}(0) = 0$ and $x(0) = 10$.

Taking the "Abaoub – Shkheam" transform of both sides of (4), we have

$$Q\left[x^{(2)}\right] + 4Q\left[x^{(1)}\right] + 4[x] = 0$$

Using the "Abaoub – Shkheam" transform for derivatives, then

$$\frac{Q(x)}{u^2s^2}-\frac{1}{u^2s}x(o)-\frac{1}{u}x^{(1)}(0)+4\frac{Q(x)}{us}-\frac{4}{u}x(o)+4Q(x)=0$$
$$Q(x)\left[\frac{1}{u^2s^2}+\frac{4}{us}+4\right]=\frac{1}{u^2s}(10)+\frac{1}{u}(0)+\frac{4}{u}(10)$$
$$Q(x)=\frac{10s+40us^2}{4u^2s^2+4us+1}$$
$$Q(x)=\frac{10s}{1+2us}+20\frac{us^2}{(1+2us)^2}$$

Take inverse the "Abaoub – Shkheam" transform, we get

$$x=10e^{-2t}+20te^{-2t}.$$

3.2 Applications to Electrical Circuits

The "Abaoub – Shkheam" transform can also be used to determine the charge on the capacitors and currents as functions of time.

Example (2)

We find the current in the circuit, an alternating $e.m.f.\, E\ sin\ \omega t$ is applied to an inductance L and a capacitance C in series. with the differential equation for the determination of the current I in the circuit is given as

$$L\frac{dI}{dt}+\frac{Q}{C}=E\sin\omega t \qquad (5)$$

Where $$I=\frac{dQ}{dt} \qquad (6)$$

Also, at $I=0=Q\ ,t=0.$

Taking the "Abaoub – Shkheam" transform of both sides of (5) and (6), we have

$$LQ[I^{(1)}]+\frac{1}{C}Q[Q]=E\,\mathrm{Q}[\sin\omega t]$$
$$Q[I^{(1)}]+\frac{1}{LC}Q[Q]=\frac{E}{L}\mathrm{Q}[\sin\omega t]$$
$$Q\left[I^{(1)}\right]+n^2Q[Q]=\frac{E}{L}\mathrm{Q}[\sin\omega t] \qquad ;n^2=\frac{1}{LC} \qquad (7)$$
$$\frac{Q(I)}{us}-\frac{1}{u}I(o)+n^2Q(Q)=\frac{E}{L}\left(\frac{\omega us^2}{1+\omega^2u^2s^2}\right) \qquad (8)$$

Apply the initial condition, then

$$\frac{Q(I)}{us}+n^2Q(Q)=\frac{E}{L}\left(\frac{\omega us^2}{1+\omega^2u^2s^2}\right)$$

$$Q(I)=Q\left(Q^{(1)}\right)=\frac{1}{us}Q(Q)-\frac{1}{u}Q(0)=\frac{1}{us}Q(Q)$$
$$Q(Q)=usQ(I) \qquad (9)$$

From (8) and (9), we get

$$Q(I)\left(\frac{1}{us}+n^2us\right)=\frac{E}{L}\left(\frac{\omega us^2}{1+\omega^2u^2s^2}\right)$$

$$Q(I)=\frac{E}{L}\left(\frac{\omega us^2}{1+\omega^2u^2s^2}\right)\left(\frac{us}{1+n^2u^2s^2}\right)$$

$$Q(I)=\frac{E}{L}\left(\frac{\omega}{n^2-\omega^2}\right)\left(\frac{s}{1+\omega^2u^2s^2}-\frac{s}{1+n^2u^2s^2}\right)$$

Take inverse the "Abaoub – Shkheam" transform, we get

$$I=\frac{E}{L}\left(\frac{\omega}{n^2-\omega^2}\right)\left[Q^{-1}\left(\frac{s}{1+\omega^2u^2s^2}\right)-Q^{-1}\left(\frac{s}{1+n^2u^2s^2}\right)\right]$$

$$I=\frac{E\omega}{L}\left(\frac{1}{n^2-\omega^2}\right)(cos\omega t-cos\omega t).$$

3.3 Applications to in nuclear physics

The fundamental relationship explaining radioactive decay is represented by this equation, where $N = N(t)$ denotes the number of undecayed atoms remaining in a sample a radioactive isotope at the time and is the decay constant.[3]

The goal of researching sequential decay is to determine the number of atoms or nuclei in each component of this chain. So, if we symbolize the number of atoms of the parent elements in time t by the N_1, and the decay constant for it is λ_1, the number of atoms of the daughter nucleus N_2, which is active, and the decay constant for it is λ_2 the number of atoms of the granddaughter nucleusN_3, which is a stable element. If the boundary condition for all atoms or nuclei at time $t = 0$ is If we assume that the rate of decay of the mother atoms is precisely equal to the rate of production of the daughter atoms and that the rate of decay of the daughter atoms equals the rate of production of the granddaughter atoms, the entire process can be represented using the three equations as follows:

$$\frac{dN_1}{dt}=-\lambda_1N_1 \quad (10)$$

$$\frac{dN_2}{dt}=\lambda_1N_1-\lambda_2N_2 \quad (11)$$

$$\frac{dN_3}{dt}=-\lambda_2N_2 \quad (12)$$

According to the Basic Law, the relationship (10) defines the rate of disintegration for the parent atoms. The equation (11) states that the daughter atoms are generated at a rate and disintegrate at a rate, whereas the equation (12) states that the granddaughter atoms N_3 are formed at a rate. [7] By solving the set of equations (10,11,12), the number of atoms of each type of the three members of the chain N_1, N_2 and N_3 as a function of time may be obtained using the "Abaoub - Shkheam" transform as follows:

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

We can use the"Abaoub – Shkheam" transform to solve this equation. Rearranging the above equation, we get

$$\frac{dN_1}{dt} + \lambda_1 N_1 = 0$$

Taking "Abaoub – Shkheam" transform on both sides

$$Q[{N_1}^{(1)}] + \lambda_1 Q[N_1] = 0$$

$$\frac{Q(N_1)}{us} - \frac{1}{u} N_1(o) + \lambda_1 Q(N_1) = 0$$

$$Q(N_1)\left(\frac{1}{us} + \lambda_1\right) = \frac{1}{u} N_1(o)$$

$$Q(N_1) = \frac{s}{1 + \lambda_1 us} N_1(o) \qquad (13)$$

Take inverse "Abaoub – Shkheam" transform, we get

$$N_1(t) = e^{-\lambda_1 t} N_1(o)$$

Now equation (11)

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Taking "Abaoub – Shkheam" transform on both sides

$$Q[{N_2}^{(1)}] + \lambda_2 Q[N_2] = \lambda_1 Q[N_1]$$

$$\frac{Q(N_2)}{us} - \frac{1}{u} N_2(o) + \lambda_2 Q(N_2) = \lambda_1 Q(N_2)$$

$$Q(N_2)\left(\frac{1}{us} + \lambda_2\right) = \lambda_1 Q(N_1) \qquad (14)$$

Since $N_2(o) = 0$

Substituting from (13) in to (14) we get

$$Q(N_2) = \lambda_1 N_1(o)\left(\frac{s}{1 + \lambda_1 us}\right)\left(\frac{us}{1 + \lambda_2 us}\right) \qquad (15)$$

$$Q(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(o)\left[\left(\frac{s}{1 + \lambda_1 us}\right) - \left(\frac{s}{1 + \lambda_2 us}\right)\right]$$

Take inverse "Abaoub – Shkheam" transform, we get

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(o)\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

Now equation (12)

$$\frac{dN_3}{dt} = -\lambda_2 N_2$$

Taking "Abaoub – Shkheam" transform on both sides

$$Q\left[{N_3}^{(1)}\right] = -\lambda_2 Q[N_2]$$

$$\frac{Q(N_3)}{us} - \frac{1}{u} N_3(o) = -\lambda_2 Q(N_2) \quad (16)$$

$$Q(N_3) = -us\,\lambda_2 Q(N_2)$$

Since $N_3(o) = 0$

Substituting from (15) in to (16)we get

$$Q(N_3) = -\lambda_1\lambda_2 N_1(o)\left[us\left(\frac{s}{1+\lambda_1 us}\right)\left(\frac{us}{1+\lambda_2 us}\right)\right]$$

$$Q(N_3) = N_1(o)\left[s - \frac{\lambda_2}{\lambda_2-\lambda_1}\left(\frac{s}{1+\lambda_1 us}\right) + \frac{\lambda_1}{\lambda_2-\lambda_1}\left(\frac{s}{1+\lambda_2 us}\right)\right]$$

Take inverse "Abaoub – Shkheam" transform, we get

$$N_3(t) = N_1(o)\left[1 - \frac{\lambda_2}{\lambda_2-\lambda_1} e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1} e^{-\lambda_2 t}\right].$$

3.4 Applications to in Mechanical Engineering

In the Mechanical engineering field, the "Abaoub – Shkheam" Transform is widely used to solve differential equations occurring in the mathematical modeling of mechanical systems to find the transfer function of that particular system [10]. Following the example describes how to use the Abaoub-Shkheam Transform to find.

Example (3)

The tank shown in the figure is initially empty $t = 0$. A constant rate of flow Q_i is added for $t > 0$. The rate at which flow leaves the tank is $Q_0 = CH$.. The tank's cross-sectional area is A. Solve the differential equation for H's head. Calculate the time constant and the system's transfer function.

Given that $Q_0 = CH$.

Let, $M =$ Mass of fluid

e =Density of fluid

Mass $= M =$ Volume density $= AH \times e$

Mass flow rate $= M = \frac{dM}{dt} = \frac{d}{dt}(AH \times e) = eA \times \frac{dH}{dt}$

We know that, Mass flow rate into tank = Mass in flow rate – Mass out flow rate.

$$eA\frac{dH}{dt} = eQ_i - eQ_0$$

$$A\frac{dH}{dt} = Q_i - Q_0$$

$$A\frac{dH}{dt} = Q_i - CH \quad : Q_0 = CH$$

$$Q_i = A\frac{dH}{dt} + CH$$

This equation represents the differential equation for head H

Now, Taking Abaoub-Shkheam Transform on both sides

$$Q[Q_i] = AQ[H^{(1)}] + CQ[H]$$

$$Q(Q_i) = A\left[\frac{Q(H)}{us} - \frac{H(0)}{u}\right] + CQ(H)$$

$$Q(Q_i) = \frac{A}{us}H(s) + CQ(H) \ ; \ H(0) = 0$$

$$Q(Q_i) = \left[\frac{A}{us} + C\right]Q(H)$$

$$\frac{Q(H)}{Q(Q_i)} = \frac{us}{A + cus} \qquad (17)$$

But $Q_0 = CH$, Taking Abaoub-Shkheam Transform, we get $Q[Q_0] = CQ[H]$

$$Q(Q_0) = CQ(H)$$

$Q(H) = \frac{Q(Q_i)}{C}$, Using this in equation (17), we get

$$\frac{Q(Q_0)}{CQ(Q_i)} = \frac{us}{A + cus}$$

$$\frac{Q(Q_0)}{Q(Q_i)} = \frac{us}{\frac{A}{C} + us}$$

$$\frac{Q(Q_0)}{Q(Q_i)} = \frac{1}{1 + \left(\frac{A}{C}\right)us}$$

This equation represents the transfer function of system. time constant $\tau = \frac{A}{C}$.

4.0 Conclusion

In this paper, we are solving ordinary differential equations in mechanics and electrical circuit problems, and Mechanical Engineering and nuclear physics, using by integral transform Abaoub-Shkheam transform. This new method is more efficient and easier to handle such differential equations in comparison with other methods. Also, this method is very efficient and simple, and engineering applications can be extended to other problems of diverse nature, and the application of the Abaoub-Shkheam Transform in the nuclear physics field, especially in the decay and growth of nuclei or atoms. and is a very effective tool for simplifying many complex problems in the nuclear physics field. The results have been obtained using very little computational work and spending very little time. Abaoub-Shkheam Transform is a very effective tool to simplify very complex problems in the area of stability and control. It goes without saying that Abaoub-Shkheam Transform is put to tremendous use in the Mechanical Engineering field.

5.0 Acknowledgement

I declare that the article has not been published or under consideration for publication elsewhere.

6.0 References

[1] Abaoub, and A. Shkheam, (2020). The New Integral Transform "Abaoub-Shkheam transform", Iaetsd journal for advanced research in applied sciences, VII. Issue (VI), June, 2394-8442.

[2] Abaoub, and A. Shkheam, 2020. Utilization Abaoub-Shkheam transform in solving linear integral equation of Volterra, International Journal of Software & Hardware Research in Engineering (IJSHRE) ISSN-2347-4890 Volume 8 Issue 12 December.

[3] Amna E. Elhag, 2Khalda T. Osman .2021, Applications of Laplace Transform in Successive Radioactive Decay of Nucleus, International Journal of Novel Research in Physics Chemistry & Mathematics Vol. 8, Issue 3, pp: (1-3), Month: September - December.

[4] Asmaa O. Mubayrash, Huda M. Khalat. (2022), New method for cryptography using abaoub- shkheam transform The Libyan Journal of Science- University of Tripoli Vol. 25, No. 02 ,35-39.

[5] Asmaa Omar Mubayrash. 2022, Solving the partial Differential Equations Using the Modified Abaoub-Shkheam Adomian Decomposition Method, ALQIRTAS JURNA, no.21, pp: 70-83.

[6] Asmaa Omar Mubayrash, Suad Mawloud Zali, zaineb alkawash. 2021, The Use of Abaoub- Shkheam Transform for Solving Partial Differential Equations, ALQIRTAS JURNA, no.20, pp: 67-74.

[7] English, A.C., Cranshaw, T.E., Demers, P., Harvey, J.A., Hincks, E.P., Jelley, J.V. and May, A.N., 1947. The (4 n+ 1) Radioactive Series. Physical Review, 72(3), p.253.

[8] Huda M. Khalat. Asmaa O. Mubayrash, (2024), A New Cryptographic Scheme Using Integral Transform and Matrix Decomposition transform The Libyan Journal of Science- University of Tripoli Vol. 27, No. 02,55-63.

[9] Kokane, A.S. and Wadkar, B.R., 2021, May. Applications of Laplace Transform in science and technology. In Journal of Physics: Conference Series (Vol. 1913, No. 1, p. 012127). IOP Publishing.

[10] L S Sawant May .2018, Applications of Laplace Transform in Engineering Fields International Research Journal of Engineering and Technology (IRJET) Volume 05 Issue: 05.

[11] P. Senthil Kumar and A. Viswanathan, 2018. "Applications of Mahgoub Transform to Mechanics, Electrical Circuit problems", Inter.Journal of Science and Research, Vol.7, No.7, pp: 195 – 197.

[12] Sawant, L.S., 2018. Applications of Laplace transform in engineering fields. Int. Res. J. Eng. Technol. (IRJET), 5(05).

[13] Tarig M. Elzaki, Salih M. Elzaki, and Elsayed A. Elnour, 2012. "Applications of New Transform Elzaki Transform to Mechanics, Electrical Circuits and Beams problems", Global Journal of Mathematical Sciences: Theory and Practical, Vol.4, No.1, pp. 25 – 34.